MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics is intended to give students an ability to solve problems, to communicate their ideas and strategies, and to apply their skills in other disciplines. Students are expected to understand and investigate mathematical concepts, to use mathematics in real-world situations, and to select and use appropriate technology to model and study mathematical processes.

Students will use mathematical methods to learn about six strands: Quantity (number), Algebraic Representation, Shape (geometry), Measurement, Chance and the Use of Data, and Mathematical Patterns. In every strand, it is important for all students to have a conceptual framework, a knowledge of procedures, a sense of reasonable results, and a confidence to apply their skills.

Content standards indicate what all students should know, understand, and be able to do in a specific content area.

Benchmarks define our expectations for students' knowledge, skills, and abilities along a developmental continuum in each content area. That continuum is focused at three points—the end of grade 4, grade 8, and grade 12. In this document, performance indicators that appear under more than one benchmark are indicated by an asterisk (*).

The following standards are for all students, and the expanded benchmarks are specifically labeled throughout the document:

Content Standard 1 - Students engage in the mathematical processes of problem solving and reasoning, estimation, communication, connections and applications, and using appropriate technology.

Content Standard 2 - Students demonstrate understanding of and an ability to use numbers and operations.

Content Standard 3 - Students use algebraic concepts, processes, and language to model and solve a variety of real-world and mathematical problems.

Content Standard 4 - Students demonstrate understanding of shape and an ability to use geometry.

Content Standard 5 - Students demonstrate understanding of measurable attributes and an ability to use measurement processes.

Content Standard 6 - The students demonstrate understanding of an ability to use data analysis, probability, and statistics.

Content Standard 7 - Students demonstrate understanding of and an ability to use patterns, relations and functions.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 1

Students engage in the mathematical processes of problem solving and reasoning, estimation, communication, connections and applications, and using appropriate technology.

Essence of Standard 1: Process.

Rationale
These processes are essential to all mathematics and must be incorporated in all other mathematics standards.

Benchmarks
For each of the benchmarks below indicators are incorporated throughout for standards 2–7. They are particularly the focus of the following benchmarks:

- **Numbers and operations 2.6**
- **Algebra 3.4**
- **Geometry 4.5**
- **Measurement 5.3 and 5.4**
- **Data, probability and statistics 6.3 and 6.5**
- **Patterns, relations, and functions 7.3**

End of Grade 4 -

Students will:

1.1 Solve problems from many contexts using a variety of strategies (e.g., estimate, make a table, look for a pattern, and simplify the problem). Explain the methods for solving these problems.

1.2 Apply estimation strategies throughout the problem-solving process.

1.3 Communicate mathematical ideas in a variety of ways (e.g., written, verbal, concrete, pictorial, graphical, algebraic).

1.4 Recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school.

1.5 Select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, and computer.

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End of Grade 8 –

Students will:

1.1 Formulate and solve multi-step and nonroutine problems using a variety of strategies. Generalize methods to new problem situations.

1.2 Select and apply appropriate estimation strategies throughout the problem-solving process.

1.3 Interpret and communicate mathematical ideas and logical arguments using correct mathematical terms and notations.

1.4 Recognize and investigate the relevance and usefulness of mathematics through applications, both in and out of school.

1.5 Select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Upon Graduation – End of Grade 12

Students will:

1.1 Recognize and formulate problems from situations within and outside mathematics and apply solution strategies to those problems.

1.2 Select, apply, and evaluate appropriate estimation strategies throughout the problem-solving process.

1.3 Formulate definitions, make and justify inferences, express generalizations, and communicate mathematical ideas and relationships.

1.4 Apply and translate among different representations of the same problem situation or of the same mathematical concept. Model connections between problem situations that arise in disciplines other than mathematics.

1.5 Select and use appropriate technology to enhance mathematical understanding. Appropriate technology may include, but is not limited to, paper and pencil, calculator, computer, and data collection devices.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 2

Students demonstrate understanding of and an ability to use numbers and operations.

Essence of Standard 2: Number concepts, concepts of operations, computing and estimating.

Rationale

An understanding of numbers and how they are used is necessary in the everyday world. Computational skills and procedures should be developed in context so the learner perceives them as tools for solving problems.

Benchmarks

End of Grade 4 –

Students will:

2.1 Exhibit connections between the concrete and symbolic representation of a problem or concept.

Expanded Benchmarks

2.1.1 Demonstrate an understanding of symbols, numbers, and operations through concrete models.

2.1.1.1* Match a numeral to a quantity of a set of objects.

2.1.1.2* Match number words and numerals with models of 2-digit numbers (e.g., with base-ten blocks).

2.1.1.3* Match number words and numerals with models of 3-digit numbers (e.g., with base-ten blocks).

2.1.1.4* Divide a whole unit (e.g., a pizza, a piece of paper) into halves, quarters, thirds.

2.1.1.5* Connect plus (+) and minus (–) symbols to operations.

2.1.1.6* Demonstrate an understanding of multiplication using concrete materials, as by (1) combining equal groups of objects (repeated addition), (2) an *m*-by-*n* array, or (3) the number of possible pairings of objects in two different sets.

2.1.1.7* Demonstrate an understanding of division using concrete materials, as by (1) sharing equally or (2) repeated subtraction.

2.2 Use the number system by counting, grouping and applying place value concepts.

Expanded Benchmarks

2.2.1 Demonstrate an understanding of whole numbers.

2.2.1.1 Demonstrate an understanding that numbers, as opposed to letters, are used to express quantity, order, or size/amount.

2.2.1.2 Demonstrate the concept of one (e.g., "Hit the switch one time"; "Give me one").
2.2.1.3 Demonstrate that a collection of objects has a quantity.
2.2.1.4 Show a quantity (e.g., "Show me four").
2.2.1.5 Apply a number (word) to a quantity of objects in a collection (e.g., "How many in this collection?").
2.2.1.6 Demonstrate an ability to ascertain quantity without counting (up to 3 or 4).
2.2.1.7 Demonstrate an understanding of the concept of some and none.
2.2.1.8 Associate the number 0 with empty sets in different settings.
2.2.1.9 Appropriately label the quantity of an empty set with words (e.g., "none," "zero," "nothing").
2.2.1.10 Demonstrate an understanding that numbers are used in different ways (e.g., quantity vs. order; 4 objects vs. 4th in line; counting number of people vs. saying their age.
2.2.1.11 Demonstrate an understanding that numbers are represented by numerals.
2.2.1.12 Discriminate between numeral and other printed symbols.
2.2.1.13 Match a number to a numeral (and vice versa).
2.2.1.14* Match a numeral to a quantity of a set of objects.
2.2.1.15 Identify number words "one" through "ten."
2.2.1.16 Match a number word to a quantity of a set of objects (and vice versa).
2.2.1.17 Match numerals to number words up to 10 (e.g., 1 → one; and vice versa).
2.2.1.18 Produce a numeral to 10.
2.2.1.19 Recognize two-digit numerals, 10–99.
2.2.1.20 Find two-digit numerals on a 100 (or 0–99) chart.
2.2.1.21 Produce a numeral to 100.
2.2.1.22 Fill in missing numerals on a 100 (or 0–99) chart.
2.2.1.23 Recognize three-digit numerals, 100–999.
2.2.1.24 Produce a numeral to 1000.
2.2.1.25 Demonstrate an understanding of even and odd numbers.

2.2.2 Demonstrate an understanding of place value.

2.2.2.1 Tell how many objects are in a set that is grouped as tens and ones (and vice versa) (e.g., 2 tens and 8 ones is 28; 34 is 3 tens and 4 ones).
2.2.2.2 Produce the grouping of up to 100 objects in sets of ten and remaining units.
2.2.2.3* Match number words and numerals with models of 2-digit numbers (e.g, with base-ten blocks).
2.2.2.4 Represent numbers up to 99 in an expanded form (and vice versa) (e.g., 82 = 80 + 2; 80 + 2 = 82).
2.2.2.5 Represent 3-digit numbers using groups of hundreds, tens, and ones (and vice versa) (e.g., 256 is 2 hundreds, 5 tens, and 6 ones; 1 hundred, 3 tens, and 4 ones is 134).
2.2.2.6* Match number words and numerals with models of 3-digit numbers (e.g., with base-ten blocks).

2.2.2.7 Represent numbers up to 999 in an expanded form (and vice versa) (e.g., 300 + 70 + 9 is the expanded form of 379).

2.2.3 Demonstrate an understanding of concepts of order.
- 2.2.3.1 Attend to a person associating numbers with order.
- 2.2.3.2 Identify first and last.
- 2.2.3.3 Indicate ordinal position (first, second, third, . . .) of various elements (e.g., person, activities) in a sequence.
- 2.2.3.4 Position numbers on a number line.
- 2.2.3.5 Arrange sets of objects, up to ten, from least to most (and vice versa).
- 2.2.3.6 Arrange numbers from least to greatest up to 100 (and vice versa).
- 2.2.3.7 Demonstrate an understanding of greater than (>), less than (<), and equals (=) symbols.
- 2.2.3.8 Supply the appropriate relation symbol (e.g., >, <, =) when given two quantities and a label.

2.2.4 Demonstrate an understanding of counting.
- 2.2.4.1 Attend to another person counting objects.
- 2.2.4.2 Count with another person.
- 2.2.4.3 Demonstrate that counting involves saying numbers.
- 2.2.4.4 Count using a sequential order of numbers (e.g., 1, 2, 3, 4; rote counting).
- 2.2.4.5 Demonstrate one-to-one correspondence among up to 12 objects and counting numbers, with no recounting (rational counting).
- 2.2.4.6 Demonstrate an understanding that the final number said when counting objects is the quantity of the set.
- 2.2.4.7 Count from 1 to 100 (rote counting).
- 2.2.4.8 Count by ones from a given number forward.
- 2.2.4.9 Skip count by twos, fives, and tens to 100.
- 2.2.4.10 Count 12 to 30 objects (rational counting).

2.3 model, explain, and use basic facts, the operations of addition and subtraction of whole numbers, and mental mathematics.

Expanded Benchmarks

2.3.1 Demonstrate an understanding of the basic concepts of addition, subtraction, multiplication, and division.
- 2.3.1.1 Attend to another person combining objects to add.
- 2.3.1.2 Attend to another person removing objects or comparing sets to subtract.
- 2.3.1.3 Demonstrate an understanding of the concepts of some/more/less/take away/all gone/no more/less.
- 2.3.1.4* Connect plus (+) and minus (–) symbols to operations.
- 2.3.1.5 Demonstrate an understanding of addition as combining collections of things/counting on.
- 2.3.1.6 Demonstrate an understanding of the concept of one more.

2.3.1.7 Model a written addition problem (e.g., 3 + 5) using sets of objects, combining the sets, and counting the objects, either counting all or counting on.
2.3.1.8 Demonstrate an understanding that addition is commutative (i.e., 5 + 7 = 7 + 5).
2.3.1.9 Demonstrate one-to-one correspondence between objects in two sets.
2.3.1.10 Demonstrate an understanding of subtraction as (1) taking away from a collection and (2) comparing collections of two different sizes.
2.3.1.11 Determine whether the numbers of identical objects in two groups are the same or different, for both equal and unequal sets, and which group has more (when arranged in the same structured presentation).
2.3.1.12 Determine whether the numbers of identical objects in two groups are the same or different, for both equal and unequal sets, and which group has more (when arranged in a random presentation).
2.3.1.13 Model a written subtraction problem (e.g., 8–3) using a set of objects, taking some away, and counting the remainder.
2.3.1.14 Demonstrate an understanding that subtraction is not commutative (i.e., 8–3 ≠ 3–8).
2.3.1.15 Demonstrate an understanding of the relationship between addition and subtraction.
2.3.1.16 Choose addition or subtraction as appropriate for the situation.

2.3.2 Demonstrate fluency in computing with whole numbers.
2.3.2.1 Employ strategies to recall simple addition facts, single-digit sums up to 10 (e.g., doubles, doubles plus 1, commutativity ["turn around facts"]).
2.3.2.2 Employ strategies to recall simple addition facts, single-digit sums up to 18 (e.g., doubles, doubles plus 1, commutativity ["turn-around facts"], visualizing ten-frames).
2.3.2.3 Demonstrate an understanding that adding 0 to any number equals the number.
2.3.2.4 Employ strategies to recall simple subtraction facts for single-digit differences from 10 (e.g., counting back; comparison/addition—add to the smaller number to get the larger one).
2.3.2.5 Employ strategies to recall simple subtraction facts for single-digit differences from 18 (e.g., counting back; comparison/addition—add to the smaller number to get the larger one; work backward—take away small "easy" numbers from the larger number to get to the smaller one).
2.3.2.6 Use the number line to find sums and differences (e.g., start at one addend and move the appropriate number of steps to the right for the second addend).
2.3.2.7 Demonstrate understanding that subtracting 0 from any number equals the number.
2.3.2.8 Use strategies to compute one- and two-digit addition problems. (For example, to add 17 and 8, combine 3 from the 8 with the 17 to get 20; then add 5 more to get 25.)

2.3.2.9 Use strategies to compute one- and two-digit subtraction problems. (For example, to subtract 27 from 36, consider how much you have to add to 27 to get to 36—3 to get to 30 and 6 more to get to 36, so you added 9.)
2.3.2.10 Use the 100 (or 0–99) chart to add or subtract 10 to/from a number.
2.3.2.11 Demonstrate a strategy for rounding numbers up or down.
2.3.2.12 Use strategies to solve addition, subtraction, multiplication, or division problems as appropriate for the situation.
2.3.2.13 Connect the plus (+), minus (–), multiplication (×), and division (/) symbols to the operations.
2.3.2.14 Use a calculator for whole-number computation.

2.3.3 Demonstrate fluency in computing and estimating with money.
2.3.3.1 Demonstrate an understanding of the differences between a coin and a bill.
2.3.3.2 Differentiate between coins by attributes (metal color, size, weight, texture).
2.3.3.3 Match coins to like coins and bills to like bills.
2.3.3.4 Demonstrate that coins and bills have value and can be exchanged for merchandise/goods/services.
2.3.3.5 Match coins and their values.
2.3.3.6 Use different coins to show equivalent amounts of money.
2.3.3.7 Determine the total value of several coins.
2.3.3.8 Match bills and their values.
2.3.3.9 Use different bill combinations to show equivalent amounts (e.g., five $1 bills equal one $5 bill).
2.3.3.10 Count out an exact amount of money.
2.3.3.11 Round an amount to the next dollar (next-dollar strategy).
2.3.3.12 Determine how much more money is needed when funds are insufficient.
2.3.3.13 Determine change when funds are more than cost.
2.3.3.14 Compute addition and subtraction problems with money.

2.4 Model and explain multiplication and division of whole numbers.

Expanded Benchmarks

2.4.1 Demonstrate an understanding of multiplication and division through concrete models.
2.4.1.1 Attend to another person modeling multiplication or division.
2.4.1.2* Demonstrate an understanding of multiplication, using concrete materials, as by (1) combining equal groups of objects (repeated addition), (2) an *m*-by-*n* array, or (3) the number of possible pairings of objects in two different sets.
2.4.1.3* Demonstrate an understanding of division, using concrete materials, as by (1) sharing equally or (2) repeated subtraction.

2.5 Model and explain part/whole relationships in everyday situations.

Expanded Benchmarks

2.5.1 Demonstrate an understanding of fractions.

2.5.1.1 Attend to another person modeling part/whole relationships.
2.5.1.2* Attend to another person demonstrating with concrete materials.
2.5.1.3 Demonstrate an understanding that fractional parts are parts of a whole unit.
2.5.1.4* Divide a whole unit (e.g., a pizza, a piece of paper) into halves, quarters, thirds.
2.5.1.5 Demonstrate that *n* equal parts of 1/*n* make the whole (e.g., four 1/4 pieces equal one whole; six 1/6 pieces equal one whole).
2.5.1.6 Produce fractional parts of a whole unit (e.g., shade in 2/3 or 3/4 of a whole shape) and vice versa (i.e., identify fractional parts of a whole (e.g., identify fraction that is shaded).
2.5.1.7 Order fractional parts (e.g., show that 1/2 is greater than 1/4).
2.5.1.8 Divide a set of discrete objects into fractional parts (e.g., 1/2, 1/4, 1/3, 1/10).
2.5.1.9 Recognize and demonstrate understanding of a symbol for a fraction, including meaning of numerator and denominator.
2.5.1.10 Demonstrate an understanding that fractional parts are relative to the size of the whole unit (1/4 of a 16-inch pizza is larger than 1/4 of a 10-inch pizza).
2.5.1.11 Combine fractional parts of a region (circle, square, rectangle) to make other fractions and the whole region (e.g., 1/4 + 1/4 = 1/2).
2.5.1.12 Find the result of taking fractional parts of a region (circle, square, rectangle) away from other fractional parts or the whole region (e.g., 3/4–1/4 = 1/2).

2.6 Solve problems, communicate, estimate, and apply appropriate technology involving numbers and operations.

Expanded Benchmarks

2.6.1 Solve Problems.

2.6.1.1 Choose correct strategies or procedures to solve a number problem.
2.6.1.2 Use methods and tools to solve a number problem, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
2.6.1.3 Enter numbers correctly on a calculator/write numbers correctly.
2.6.1.4 Carry out a strategy to solve a number problem.
2.6.1.5 Determine whether results make sense.

2.6.2 Estimate.

2.6.2.1 Attend to another person estimating an amount in a given set.
2.6.2.2 Use a quantitative label when making a guess (e.g., a few, many, seventeen).

2.6.2.3 Determine which of two numbers is closer to the quantity in a given set. (For example, “Is the number of objects in this set closer to 10 or to 20?” “Is 67 closer to 60 or 70?”)
2.6.2.4 Identify a reasonable quantity when guessing the amount in a given set.
2.6.2.5 Round numbers to the nearest 10 (e.g., 27 rounds to 30) or nearest 100.
2.6.2.6 Estimate sums by rounding.
2.6.2.7 Use estimation to determine whether the solution to a computational problem is reasonable.
2.6.2.8 Evaluate an estimate by comparing with the actual number.

2.6.3 Communicate.
2.6.3.1* Explain/show reasoning.
2.6.3.2* Explain the procedure for solving a number problem.

End of Grade 8 –

Student will:

2.1 Use the four basic operations with whole numbers, fractions, decimals, and integers.

2.2 Use mental mathematics and number sense in using order of operations, and order relations for whole numbers, fractions, decimals, and integers.

2.3 Use the relationships and applications of ratio, proportion, percent, and scientific notation.

2.4 Develop and apply number theory concepts (e.g., primes, factors and multiples) in real-world and mathematical problem situations.

Upon Graduation – End of Grade 12

Student will:

2.1 Use and understand the real number system, its operations, notations, and the various subsystems.

2.2 Use definitions and basic operations of the complex number system.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 3

Students use algebraic concepts, processes, and language to model and solve a variety of real-world and mathematical problems.

Essence of Standard 3: Algebraic symbols, models, and change.

Rationale

Algebra is the language of mathematics and science. Through the use of variables and operations, algebra allows students to form abstract models from contextual information.

Benchmarks

End of Grade 4 –

Students will:

3.1 Use symbols (e.g., boxes or letters) to represent numbers in simple situations.

Expanded Benchmarks

3.1.1 Demonstrate the ability to use algebraic symbols to represent and analyze situations.

3.1.1.1 Attend to another person setting up a number sentence with a box as a placeholder.

3.1.1.2 Recognize that a box is used as a placeholder in a number sentence.

3.1.1.3* Find a simple missing addend represented by a box in a number sentence.

3.1.1.4* Supply the missing number represented by a box in a number sentence, in which the operation might be addition, subtraction, or multiplication.

3.2 Explore the use of variables and open sentences to express relationships (e.g., missing addend).

Expanded Benchmarks

3.2.1 Demonstrate an understanding of change in a variety of situations.

3.2.1.1* Attend to another person showing relationships between two variables using objects (e.g., 1 can of concentrate goes with 3 cans of water to make juice; 2 cans of concentrate go with 6 cans of water), pictures, symbols, or numbers.

3.2.1.2* Recognize a cause-effect relationship between two elements (e.g., when a switch is hit, a number appears on a screen).

3.2.1.3* Demonstrate/communicate what the relationship is between two elements.

3.2.1.4* Predict how change in one element may change the other element (e.g., Increase in the number of hits of a switch increases the number of numbers that appear on a screen).

3.2.1.5* Show a relationship between two variables, using ordered pairs or a table (e.g., 1 student, 2 cookies; 2 students, 4 cookies; 3 students, 6 cookies); then make a table.
3.2.1.6* Explain the relationship between two variables (e.g., twice as many cookies as students are needed)
3.2.1.7* Attend to charts, graphs, or tables.
3.2.1.8* Given a table showing values of two variables, a box and a triangle, tell the relationship between them. For example:

△	1	2	3	4
□	4	5	6	7

The relationship is "add 3 to box to get triangle" or "box + 3 = triangle."

3.3 Use inverse operations and other strategies to solve number sentences.

Expanded Benchmarks

3.3.1 Demonstrate the ability to solve number sentences for an unknown number.
3.3.1.1 Attend to another person solving a number sentence for a missing number.
3.3.1.2* Find a simple missing addend represented by a box in a number sentence.
3.3.1.3* Supply the missing number represented by a box in a number sentence, in which the operation might be addition, subtraction, or multiplication.
3.3.1.4 Find values that satisfy an inequality (e.g., >5).
3.3.1.5 Given a numerical relationship between two variables, find the value of one given the other. (For example, if the second variable is 3 more than the first, and the first is 4, then the second is 7.)
3.3.1.6 Use or extend a T-table to find the value of a variable.

3.4.1 Solve problems, communicate, estimate, make connections, and apply appropriate technology involving algebra.

Expanded Benchmarks

3.4.1.1 Choose correct strategies or procedures to solve an algebraic problem in algebra.
3.4.1.2 Use methods and tools to solve a problem, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
3.4.1.3 Carry out a strategy to solve an algebraic problem.
3.4.1.4* Determine whether results make sense.
3.4.1.5* Explain/show reasoning.
3.4.1.6* Explain/show the procedure for solving an algebraic problem.
3.4.1.7* Explain decisions based on models, tables, or graphs.
3.4.1.8* Connect mathematical ideas.

End of Grade 8 –

Students will:

3.1 Understand the concepts of variable, expression, and equation.

3.2 Represent situations and number patterns using tables, graphs, verbal rules, equations, and models.

3.3 Recognize and use the general properties of operations (e.g., the distributive property).

3.4 Solve linear equations using concrete, numerical, and algebraic methods.

3.5 Investigate inequalities and nonlinear relationships informally.

Upon Graduation – End of Grade 12

Students will:

3.1 Use algebra to represent patterns of change.

3.2 Use basic operations with algebraic expressions.

3.3 Solve algebraic equations and inequalities: linear, quadratic, exponential, logarithmic, and power.

3.4 Solve systems of algebraic equations and inequalities, including the use of matrices.

3.5 Use algebraic models to solve mathematical and real-world problems.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 4

Students demonstrate understanding of shape and an ability to use geometry.

Essence of Standard 4: Two- and three-dimensional geometric shapes, coordinate systems, symmetry and transformations, and visual and spatial reasoning.

Rationale
The study of geometry helps students represent and make sense of the world by discovering relationships and developing spatial sense.

Benchmarks
Note: Throughout this section, "physical shapes" refers to simple shapes in manipulative sets, such as the circles, squares, triangles, rectangles, ovals, cubes, cylinders, spheres, box-shapes (rectangular prisms), pyramids, cones. "Shapes" refers to either physical or pictured shapes.

End of Grade 4 –

Students will:

4.1 Describe, model, and classify two- and three-dimensional shapes.

Expanded Benchmarks

4.1.1 Demonstrate an understanding of two- and three-dimensional geometric shapes and the relationships among them.

4.1.1.1 Attend to objects or pictures that represent 2- and 3-dimensional shapes.
4.1.1.2* Attend to another person demonstrating with concrete materials.
4.1.1.3 Touch and move objects (including use of a computer) that represent 2- and 3-dimensional shapes.
4.1.1.4 Recognize 2-dimensional physical shapes as being the same (congruent) or different.
4.1.1.5 Sort 2-dimensional physical shapes according to their shape.
4.1.1.6 Identify (name) shapes as circles, squares, triangles, rectangles, and ovals.
4.1.1.7 Match 2-dimensional physical shapes to pictures of the shapes in the same or different orientations.
4.1.1.8 Match shapes (circle, square, triangle, rectangle, oval) to like shapes in the same orientation.
4.1.1.9 Identify the position of an object relative to other objects (e.g., on, inside, outside, on top of, over, under, in front of, behind, beside).
4.1.1.10 Recognize properties of 2-dimensional shapes (e.g., having sides and corners or angles, being round or curved).
4.1.1.11 Identify circles, squares, triangles, ovals, and rectangles regardless of their orientation or general shape. (e.g., recognize equilateral, isosceles, right, and obtuse triangles as triangles, regardless of their orientation).

4.1.1.12 Produce (construct) 2-dimensional shapes. (e.g., given straws of different lengths, make a triangle, square, and rectangle).
4.1.1.13 Differentiate right angles from other angles.
4.1.1.14 Identify the defining properties of triangles, squares, and rectangles (e.g., all triangles have three sides and three angles).
4.1.1.15 Sort 3-dimensional physical shapes according to their shape.
4.1.1.16 Identify (name) shapes as cubes, cylinders, spheres, box-shapes (rectangular prisms), pyramids, and cones.
4.1.1.17 Match 3-dimensional physical shapes to pictures of the shapes.
4.1.1.18 Identify and sort cubes, cylinders, spheres, box-shapes (rectangular prisms), pyramids, and cones, regardless of their orientation or general shape.
4.1.1.19 Recognize properties of 3-dimensional shapes (e.g., having faces, edges, and corners, having flat or curved surfaces).
4.1.1.20 Recognize 2-dimensional shapes in relation to 3-dimensional shapes (e.g., face of a cube is square).
4.1.1.21 Find various shapes in the environment.

4.2 Investigate and predict results of combining, subdividing, and changing shapes.

Expanded Benchmarks

4.2.1 Demonstrate an understanding of combining and subdividing shapes.
4.2.1.1 Attend to another person combining and subdividing shapes.
4.2.1.2 Touch and move shapes (including use of a computer) toward creating new shapes.
4.2.1.3 Put shapes together to form other shapes (e.g., use two squares to form a rectangle).
4.2.1.4 Subdivide geometric shapes to form other shapes (e.g., divide a square into 2 triangles).

4.3 Identify lines of symmetry, congruent and similar shapes, and positional relationships.

Expanded Benchmarks

4.3.1 Demonstrate an understanding of symmetry, congruence, and transformations.
4.3.1.1 Attend to another person demonstrating lines of symmetry.
4.3.1.2 Identify shapes as symmetric or nonsymmetric.
4.3.1.3 Recognize and produce lines of symmetry for symmetric shapes.
4.3.1.4 Complete a symmetric figure, given the portion of the figure on one side of the line of symmetry.
4.3.1.5 Attend to another person demonstrating congruence.
4.3.1.6 Match shapes to congruent shapes in different orientations.
4.3.1.7 Use slides, flips, and turns to determine that physical shapes are congruent (match each other) and describe the motion using the terms "*slide," "flip,"* and "*turn."*
4.3.1.8 Attend to another person demonstrating transformations of shapes, e.g., sliding, rotating, flipping.

4.3.1.9 Recognize pictures of shapes resulting from slides, flips, or turns of a given shape.

4.3.2 Demonstrate an understanding of coordinate systems.
- 4.3.2.1 Follow navigational directions (e.g., to the right/left, up/down).
- 4.3.2.2 Create navigational directions. (For example, using words such as "right/left," "up/down," describe the position of a star on a grid relative to a square on the same grid.)
- 4.3.2.3 Attend to graphs or maps.
- 4.3.2.4 Locate a position on a grid/map, given letters on one axis and numbers on the other.
- 4.3.2.5 Identify (name) the coordinates of a position on a grid/map, given letters on one axis and numbers on the other.
- 4.3.2.6 Use/read/create a map to represent space (e.g., map of a classroom or playground).

Expanded Benchmarks

4.4.1 Students demonstrate an ability to perform visual and spatial reasoning.

4.4.1.1 Recall shapes and their relative positions after they have been viewed for only a brief period of time.
4.4.1.2 Recognize the relationship between a folded paper with cutouts and the appearance of that paper after it has been unfolded.
4.4.1.3 Cover a figure with shapes. (For example, given a hexagon and multiple copies of appropriately sized triangles, trapezoids, parallelograms, and other shapes, reason that the hexagon can be covered by the triangles, the trapezoids, or the parallelograms.)

4.5.1 Solve problems, communicate, estimate, make connections, and apply appropriate technology involving geometry.

4.5.1.1 Choose correct strategies or procedures to solve a geometric problem.
4.5.1.2 Use methods and tools to solve a geometric problem, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
4.5.1.3 Carry out a strategy to solve a geometric problem.
4.5.1.4* Determine whether results make sense.
4.5.1.5* Explain/show spatial reasoning.
4.5.1.6* Explain/show procedure for solving a geometric problem.
4.5.1.7* Connect mathematical ideas.

End of Grade 8 –

Students will:

4.1 Identify, describe, construct, and compare plane and solid geometric figures.

4.2 Understand and apply geometric properties and relationships (e.g., the Pythagorean Theorem).

4.3 Represent geometric figures on a coordinate grid.

4.4 Explore properties and transformations of geometric figures.

4.5 Use geometry as a means of describing the physical world.

Upon Graduation – End of Grade 12

Students will:

4.1 Construct, interpret, and draw three-dimensional objects.

4.2 Classify figures in terms of congruence and similarity and apply these relationships.

4.3 Translate between synthetic and coordinate representations.

4.4 Deduce properties of figures using transformations, coordinates, and vectors in problem solving.

4.5 Apply trigonometric ratios (sine, cosine and tangent) to problem situations involving triangles.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 5

Students demonstrate understanding of measurable attributes and an ability to use measurement processes.

Essence of Standard 5: Concepts and processes of measurement, tools, procedures, and formulas of measurement.

Rationale
The first step in scientific investigation is understanding the measurable attributes of objects.

Benchmarks

End of Grade 4 –

Students will:

5.1 Estimate, measure, and investigate length, capacity, weight, mass, area, volume, time, and temperature.

Expanded Benchmarks

5.1.1 Demonstrate facility with the tools, procedures, and formulas of length.
- 5.1.1.1 Attend to another person estimating length.
- 5.1.1.2 Attend to another person measuring length.
- 5.1.1.3 Use nonstandard units (e.g., paper clip, hand, foot) to measure the length of an object or a distance.
- 5.1.1.4 Use rulers to measure objects that are a whole number of inches or centimeters long.
- 5.1.1.5 Use a scale in which a small unit represents a large unit of length.
- 5.1.1.6 Make a reasonable estimate of a length or a distance relative to a nonstandard unit (e.g., paper clip, hand, foot).

5.1.2 Demonstrate facility with the tools, procedures, and formulas of capacity.
- 5.1.2.1 Attend to another person measuring capacity.
- 5.1.2.2 Use nonstandard tools and units (e.g., identical paper cups) to determine the capacity of a container.
- 5.1.2.3 Use standard tools (e.g., measuring cups) and standard units of capacity (e.g., tablespoons, cups, liters) to measure the capacity of a container.
- 5.1.2.4 Make reasonable estimates of the number of identical objects a container can hold (e.g., the approximate number of small balls in a jar).

5.1.3 Demonstrate facility with the tools, procedures, and formulas of weight.
- 5.1.3.1 Attend to another person weighing objects.

5.1.3.2 Use a balance scale and nonstandard weights (e.g., paper clips, marbles) to weigh objects.
5.1.3.3* Use a balance scale and standard weights (e.g., grams, ounces) to weigh objects.
5.1.3.4* Use a standard scale to weigh objects.

5.1.4 Demonstrate facility with the tools, procedures, and formulas of temperature.
5.1.4.1 Attend to another person reading temperature.
5.1.4.2* Read temperatures from a thermometer to the accuracy of the labeled numbers.

5.1.5 Demonstrate facility with the tools, procedures, and formulas of time.
5.1.5.1 Attend to another person telling time.
5.1.5.2* Tell time to the hour using an analog clock.
5.1.5.3* Tell time to the half hour using an analog clock.
5.1.5.4* Read time using a digital clock (e.g., "It is two twenty-five").

5.2 Develop the process of measuring and concepts related to units of measurement, including standard units (English and metric) and nonstandard units.

Expanded Benchmarks

5.2.1 Demonstrate an understanding of general measurement concepts.
5.2.1.1 Attend to another person using nonstandard and standard units of measurement.
5.2.1.2 Discriminate among sizes of similar objects using such words as "bigger," "smallest," "larger."
5.2.1.3 Attend to others using measurement language (e.g., longer, shorter, inch, centimeter, foot, pounds, kilograms, quart, degrees Fahrenheit).
5.2.1.4 Identify tools associated with measurement (e.g., rulers, tape measures, scales, measuring cups, thermometers, clocks).
5.2.1.5 Select the appropriate type of unit to be used in making a measurement (e.g., select among centimeter, liter, and kilogram).
5.2.1.6 Demonstrate an understanding of conservation of length, weight, and capacity/volume. (That is, when objects are moved or rearranged, their length/weight/capacity/volume remains the same.)
5.2.1.7 Demonstrate an understanding that it takes fewer larger units than smaller units to cover the same distance/space (e.g., fewer 4-in. × 6-in. cards than 3-in. × 5-in. cards cover a table; fewer adult strides than child strides measure across a room).

5.2.2 Demonstrate an understanding of the concepts and processes of length.
5.2.2.1 Use words to describe the length of objects (e.g., long, longer than, short, shortest).
5.2.2.2 Use words to compare distances or lengths (e.g., farther than, nearer than, shorter, longer, same).
5.2.2.3 Recognize an inch, foot, yard, centimeter, and meter as units for measuring length.

5.2.2.4 Use an appropriate standard unit of measurement for measuring length (e.g., centimeter vs. meter; inch vs. foot vs. yard).

5.2.3 Demonstrate an understanding of the concepts and processes of capacity.
5.2.3.1 Use words to describe and compare the amount of substances in containers (e.g., full, empty, a little, a lot, the same as, more than, less than).
5.2.3.2 Recognize a teaspoon, tablespoon, cup, pint, quart, gallon, milliliter, and liter as units for measuring capacity.
5.2.3.3 Use an appropriate standard unit of measurement for measuring capacity (e.g., teaspoon vs. cup vs. quart; milliliter vs. liter).
5.2.3.4 Recognize that the height of a substance in each of the two containers does not necessarily determine which container holds more of that substance.
5.2.3.5 Convert units within one system (e.g., 12- in. = 1 ft.; 3 ft. = 1 yd.; 100 cm = 1 m).

5.2.4 Demonstrate an understanding of the concepts and processes of weight.
5.2.4.1 Use words to describe weights of objects (e.g., heavy, heavier than, light, lightest, same weight as).
5.2.4.2 Use words to compare weights (i.e., heavier than, lighter than, or the same weight).
5.2.4.3 Recognize an ounce, pound, gram, and kilogram as units for measuring weight.
5.2.4.4 Use an appropriate standard unit of measurement for measuring the weight of an object (e.g., ounce vs. pound; gram vs. kilogram).

5.2.5 Demonstrate an understanding of the concepts and processes of temperature.
5.2.5.1 Recognize changes in temperature.
5.2.5.2 Use words to describe and compare temperatures (e.g., hot, warmer than, cooler than, cold).
5.2.5.3 Recognize degrees Fahrenheit and degrees Celsius as units for measuring temperature.
5.2.5.4 Associate certain temperature readings with hot and cold and with clothing and activities that are appropriate for certain temperatures, and vice versa.
5.2.5.5 Recognize the role of the numbers on a thermometer in measuring temperature.

5.2.6 Demonstrate an understanding of the concepts and processes of time.
5.2.6.1 Show an awareness of time relative to a sequence of events that relates to daily life.
5.2.6.2 Show an awareness of time-related symbols (e.g., pointing to a clock, a calendar, a picture of the Sun to indicate daytime).
5.2.6.3 Recognize in general terms when events in the established daily routine occur (e.g., time to wake up, time to brush teeth, time for lunch).
5.2.6.4 Sequence events by the order in which they occur or have occurred.

5.2.6.5 Demonstrate an awareness that a routine has been changed as well as the ability to follow the changes.
5.2.6.6 Understand and use gross terms to describe time events (e.g., day–night, morning–afternoon–evening, today–tomorrow–yesterday, before–after–now).
5.2.6.7 Use words to describe and compare lengths of time (e.g., long time, longer time than, less time than, shortest/least time).
5.2.6.8 Recognize minutes, hours, days, months, and years as units for measuring time.
5.2.6.9 Associate events in the established daily routine with the approximate time that they occur (e.g., use a chart depicting a daily schedule).
5.2.6.10 Recognize the sequence of the days of the week (e.g., Sunday, Monday, Tuesday).
5.2.6.11 Name the current day of the week, yesterday, and tomorrow. (For example, if today is Tuesday, then yesterday was Monday, and tomorrow is Wednesday.)
5.2.6.12 Distinguish between weekdays and the weekend.
5.2.6.13 Recognize the relationship of the calendar to days, weeks, and months.
5.2.6.14 Locate days of the week and dates on a calendar (e.g., birthday, holidays, today's date).

5.3 Apply measurement skills to everyday situations.

Expanded Benchmarks

5.3.1 Solve problems, communicate, estimate, make connections, and apply appropriate technology involving measurement.

5.3.1.1 Attend to a real-world problem that requires measurement.
5.3.1.2 Attend to another person setting up a measurement problem or handling materials to be measured.
5.3.1.3 Choose correct strategies or procedures to solve a measurement problem.
5.3.1.4 Use methods and tools to solve a measurement problem, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
5.3.1.5 Measure correctly.
5.3.1.6 Carry out a strategy to solve a measurement problem.
5.3.1.7 Determine whether results make sense (e.g., evaluate an estimate or a solution to a problem).
5.3.1.8* Explain/show reasoning.
5.3.1.9* Explain/show the procedure for solving a measurement problem.

5.4 Select and use appropriate tools and techniques.

Expanded Benchmarks

5.4.1 Demonstrate an ability to use measurement tools.

5.4.1.1 Select the appropriate tool to be used in making a measurement.

5.4.1.2* Use rulers to measure objects that are a whole number of inches or centimeters long.
5.4.1.3* Use standard tools (e.g., measuring cups) and standard units of capacity (e.g., tablespoons, cups, liters) to measure the capacity of a container.
5.4.1.4* Use a balance scale and standard weights (e.g., grams, ounces) to weigh objects.
5.4.1.5* Use a standard scale to weigh objects.
5.4.1.6* Read temperatures from a thermometer to the accuracy of the labeled numbers.
5.4.1.7* Tell time to the hour using an analog clock.
5.4.1.8* Tell time to the half hour using an analog clock.
5.4.1.9* Read time using a digital clock (e.g., “It is two twenty-five”).

End of Grade 8 –

Students will:

5.1 Estimate, make, and use measurements to describe, compare, and/or contrast objects in real-world situations.

5.2 Select and use appropriate units and tools to measure to a level of accuracy required in a particular setting.

5.3 Apply the concepts of perimeter, area, volume and capacity, weight and mass, angle measure, time, and temperature.

5.4 Demonstrate understanding of the structure and use of systems of measurement, including English and metric.

5.5 Use the concepts of rates and other derived and indirect measurements.

5.6 Demonstrate relationships between formulas and procedures for determining area and volume.

Upon Graduation – Ends of Grade 12

Students will:

5.1 Apply concepts of indirect measurements (e.g., using similar triangles to calculate a distance).

5.2 Use dimensional analysis to check reasonableness of procedures.

5.3 Investigate systems of derived measures (e.g., km/sec, g/cm^3).

5.4 Apply the appropriate concepts of estimates in measurement, error in measurement, tolerance, and precision.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 6

The students demonstrate understanding of and an ability to use data analysis, probability, and statistics.

Essence of Standard 6: Collecting, organizing, and displaying data, statistical methods, conclusions and inferences based on data, probability.

Rationale

With society's expanding use of data for prediction and decision making, it is important that students develop an understanding of the concepts and processes used in analyzing data.

Benchmarks

End of Grade 4 –

Students will:

6.1 Collect, organize, and display data.

Expanded Benchmarks

6.1.1 Demonstrate facility in collecting, organizing, and displaying data.

6.1.1.1 Attend to another person collecting and recording data.
6.1.1.2* Attend to another person demonstrating with concrete materials.
6.1.1.3 Indicate an awareness of collections within the environment (e.g., books, shoes, boys, girls).
6.1.1.4 Add to collections.
6.1.1.5 Given a class of objects, sort into categories and subcategories (e.g., laundry into shirts, pants, socks, and then each category into colors).
6.1.1.6 When given a problem or situation, determine the data that must be collected.
6.1.1.7 Identify where to obtain data about a category.
6.1.1.8 Identify how to obtain data about a category.
6.1.1.9 Use symbols to represent data (e.g., marks on a page, tokens).
6.1.1.10 Gather data about students (e.g., number of pockets each student has in clothing) or one's school.
6.1.1.11 Determine which questions to ask to gain information.
6.1.1.12 Sort data into general and subcategories to solve a problem or describe situation (e.g., how many students have hats, how many boys have hats).
6.1.1.13* Attend to charts, graphs, or tables.
6.1.1.14 Display data using concrete objects and then pictures of the objects.
6.1.1.15 Display data using abstract representations (e.g., tally marks).
6.1.1.16 Demonstrate an understanding that data can be displayed in a variety of ways.
6.1.1.17 Set up a graph (i.e., label axes, provide title).

6.1.1.18 Use simple tables, charts, or graphs to represent meaningful information.
6.1.1.19 Create a simple graph, frequency plot, or frequency table using real objects and/or symbols.
6.1.1.20 Display two or more categories on a bar graph.
6.1.1.21 Compare table and bar graph presentations of data.

6.2 Construct, read, and interpret displays of data, including graphs.

Expanded Benchmarks

6.2.1 Demonstrate an understanding of statistical methods.

6.2.1.1 Attend to a display of data.
6.2.1.2 Describe features of the data (e.g., number of pockets in students' clothing today ranged from 0 to 6).
6.2.1.3 Rearrange data in a variety of ways to answer different questions. (For example, regarding a favorite ice cream graph: How many like each flavor? What is the favorite flavor? How many like a flavor other than the favorite one?)
6.2.1.4 Determine which category has the most/least.
6.2.1.5 Determine which data point is in the middle in appropriate data sets.

6.3. Formulate and solve problems that involve collecting and analyzing data.

Expanded Benchmarks

6.3.1 Demonstrate the ability to draw conclusions and make inferences based on data.

6.3.1.1 Attend to a real-world problem that requires collecting and analyzing data.
6.3.1.2 Recognize the use of comparison words to describe collections in the school setting, (e.g., more/fewer/same/none/larger/smaller/less/most).
6.3.1.3 Use words to describe collections in the school setting, (e.g., more/fewer/same/none/larger/smaller/less/most).
6.3.1.4 Describe the characteristics of categories and subcategories of data using comparison words (e.g., lots of boys wear hats to school and some girls wear hats to school).
6.3.1.5 Compare categories of data using comparison words (e.g., more boys than girls wear hats to school).
6.3.1.6 Make decisions based on data, a table or a graph.
6.3.1.7 Choose correct strategies or procedures to solve a data, statistics, or probability problem.
6.3.1.8 Use methods and tools to solve a data, statistics, or probability problem, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
6.3.1.9 Gather, display and analyze data correctly.
6.3.1.10 Carry out a strategy to solve a data, statistics, or probability problem.
6.3.1.11* Determine whether results make sense.

6.4 Demonstrate basic concepts of chance (e.g., equally likely events, simple probabilities).

Expanded Benchmarks

6.4.1 Demonstrate an understanding of probability.

6.4.1.1 Attend to another person using a chance device (e.g., spinner, dice) and to a person recording outcomes of a chance device.
6.4.1.2 Display interest in a chance device.
6.4.1.3 When given one set of one object and another set of two objects, describe possible combinations.
6.4.1.4 When given two sets of limited numbers of items, form distinct combinations.
6.4.1.5 Demonstrate that a change in the number of objects in one category affects the outcome of combining that category with another.
6.4.1.6 Give all possible outcomes for a given chance device. (For example, a die can turn up 1, 2, 3, 4, 5, or 6.)
6.4.1.7 Attend to such prediction language as "likely," "not likely," "equally likely."
6.4.1.8 Predict the outcome of a chance event using a chance device (e.g., tossing coin, rolling die, spinning spinner).
6.4.1.9 Collect and record outcomes using chance devices.
6.4.1.10 Describe the frequency of occurrences for a chance device.
6.4.1.11 Use outcome information to predict future occurrences.
6.4.1.12 Design a spinner given the probabilities of outcomes. (For example, a spinner where red and blue are equally likely to be the outcome would be half red and half blue.)
6.4.1.13 Demonstrate an understanding that actual outcomes are based on the probability of an event occurring rather than on extraneous factors. (For example, a spinner that is 3/4 blue and 1/4 red is more likely to come up blue, even if the student's favorite color is red.)
6.4.1.14 Determine whether a game of chance is fair. (For example, using the spinner described in 6.4.1.13, if one player gets a point if the blue is spun and the other gets a point if the red is spun, is the game fair?)

Expanded Benchmarks

6.5.1 Communicate, estimate, make connections, and apply appropriate technology involving data and probability.

6.5.1.1 Communicate the relationships between categories of collected data.
6.5.1.2 Explain/show how decisions were made, using a table or graph.
6.5.1.3 Explain reasoning in solving a data, statistics, or probability problem.
6.5.1.4 Make estimates about data as appropriate.
6.5.1.5* Connect mathematical ideas.

End of Grade 8 –

Students will:

6.1 Systematically collect, organize, and describe data.

6.2 Construct, read, and interpret tables, charts, and graphs.

6.3 Draw inferences, construct, and evaluate arguments based on data analysis and measures of central tendency.

6.4 Construct sample spaces and determine the theoretical and experimental probabilities of events.

6.5 Make predictions based on experimental results or probabilities.

Upon Graduation – End of Grade 12

Students will:

6.1 Use curve fitting to make predictions from data.

6.2 Apply measures of central tendency and demonstrate understanding of the concepts of variability and correlation.

6.3 Select an appropriate sampling method for a given statistical analysis.

6.4 Use experimental probability, theoretical probability, and simulation methods to represent and solve problems, including expected values.

6.5 Design a statistical experiment to study a problem and communicate the outcomes.

6.6 Describe, in general terms, the normal curve and use its properties to answer questions about sets of data that are assumed to be normally distributed.

MONTANA STANDARDS AND EXPANDED BENCHMARKS FOR MATHEMATICS

Mathematics Content Standard 7

Students demonstrate understanding of and an ability to use patterns, relations and functions.

Essence of Standard 7: Patterns, relations and functions.

Rationale

One of the central themes of mathematics is the study of patterns, relations, and functions. Exploring patterns helps students develop mathematical power and instills in them an appreciation for the beauty of mathematics.

Benchmarks

End of Grade 4 –

Students will:

7.1 Recognize, describe, extend, and create a variety of patterns.

Expanded Benchmarks

7.1.1 Demonstrate an understanding of patterns.

7.1.1.1 Attend to another person making patterns and to a person describing patterns.
7.1.1.2 Attend to another person demonstrating with concrete materials.
7.1.1.3 Display interest in manipulatives for making patterns.
7.1.1.4 Recognize and anticipate an event that occurs repeatedly (e.g., repeated ringing of a bell).
7.1.1.5 Reproduce (match) a repeated event.
7.1.1.6 Recognize and indicate when a change has interrupted a regular event (e.g., a drumbeat between bell rings).
7.1.1.7 Reproduce (match) a continuing pattern of a single object, shape, design or number. (For example, a pattern such as 3, 3, 3, 3 is displayed, and student produces the same pattern right below it.)
7.1.1.8 Extend and explain a continuing pattern of a single object, shape, design, or number by adding on the next object, shape, design, or number (e.g., circle, circle, circle).
7.1.1.9 Reproduce (match) an alternating pattern of two or more objects, shapes, or numbers. (For example, a pattern such as 2, 3, 2, 3, 2, 3 is displayed and student produces the same pattern right below it.)
7.1.1.10 Extend and explain an alternating pattern of two or more objects, shapes, designs, or numbers (e.g., circle, square, circle, square, circle. . .).
7.1.1.11 Extend or supply a missing element in a repeating pattern by attribute or number (e.g., circle, square, triangle, circle, square, triangle, circle, _____, triangle).
7.1.1.12 Create a repeating pattern using objects, shapes, designs, or numbers.

7.1.1.13 Reproduce (match) a growing pattern by attribute or number (e.g., circle, square, circle, circle, square, circle, circle, circle, square. . .).
7.1.1.14 Use counting as a strategy to extend a number pattern (e.g., 5, 10, 15, 20, ___, ___).
7.1.1.15 Extend a growing pattern by supplying the next element by attribute or number (e.g., 3, 7, 11, 15, ___, ___).
7.1.1.16 Create a growing pattern by attribute or number.
7.1.1.17 Group/sort objects into sets (e.g., big buttons, little buttons).
7.1.1.18 Use collections to make patterns (e.g., cup with 2 buttons, cup with 4 buttons, cup with 6 buttons . . .).
7.1.1.19 Find patterns in common configurations such as calendars, number lines, and 100 charts.
7.1.1.20 Continue an established pattern within a configuration such as a calendar, number line, or 100 chart (e.g., on a calendar, continue patterns of numbers, days of week).
7.1.1.21 Locate a pattern in order to solve a problem. (For example, recall a phone number by remembering that there is a pattern, such as in 555-1212.)
7.1.1.22 Describe a pattern used to solve a problem.

7.1.2 Demonstrate an understanding of relations and functions.

7.1.2.1* Attend to another person showing relationships between two variables using objects (e.g., 1 can of concentrate goes with 3 cans of water to make juice; 2 cans of concentrate go with 6 cans of water), pictures, symbols, or numbers.
7.1.2.2* Recognize a cause/effect relationship between two elements (e.g., when a switch is hit, a number appears on a screen).
7.1.2.3* Demonstrate/communicate the nature of the relationship between two elements.
7.1.2.4* Predict how change in one element may change the other element (e.g., Increase in the number of hits of a switch increases the number of numbers that appear on a screen).
7.1.2.5* Show a relationship between two variables, using ordered pairs or a table (e.g., 1 student, 2 cookies; 2 students, 4 cookies; 3 students, 6 cookies); then make a table.
7.1.2.6* Explain the relationship between two variables (e.g., you need twice as many cookies as students are needed).
7.1.2.7* Attend to charts, graphs, or tables.
7.1.2.8* Given a table showing values of two variables, a box and a triangle, tell the relationship between them.
For example:

△	1	2	3	4
□	4	5	6	7

The relationship is “add 3 to box to get triangle” or “box + 3 = triangle.”

7.2 Represent and describe mathematical and real-world relationships.

Expanded Benchmarks

7.2.1 Demonstrate the ability to create models to represent mathematical relationships.

7.2.1.1 Attend to another person modeling mathematical relationships (e.g., modeling different numbers).
7.2.1.2 Display interest in mathematical models.
7.2.1.3* Model sets that contain nothing or one or more items (some, none).
7.2.1.4* Demonstrate that objects defined by a shared attribute form a set to which a number can be applied. (For example, Make a set of red triangles. How many are there?)
7.2.1.5* Model sets of the same/different amounts and compare them.
7.2.1.6* Model addition and subtraction situations in story problems using objects or pictures.
7.2.1.7 Model mathematical problems.
7.2.1.8* Attend to charts, graphs, or tables.
7.2.1.9 Use models, tables, and graphs to make decisions.

Expanded Benchmarks

7.3.1 Solve problems, communicate, estimate, make connections, and apply appropriate technology involving patterns, relations, and functions.

7.3.1.1 Choose correct strategies or procedures to solve a problem in patterns, relations, or functions.
7.3.1.2 Use methods and tools to solve a problem involving patterns, relations, or functions, including drawing pictures, modeling with objects, estimating, using paper and pencil, and using a calculator.
7.3.1.3 Carry out a strategy to solve problems involving patterns, relations, or functions.
7.3.1.4* Determine whether results make sense.
7.3.1.5* Explain/show reasoning.
7.3.1.6 Explain/show the procedure for solving a problem involving patterns, relations, or functions.
7.3.1.7* Connect mathematical ideas.

End of Grade 8 –

Students will:

7.1 Describe, extend, analyze, and create a variety of patterns and functions.

7.2 Describe and represent relationships with tables, graphs, and rules.

7.3 Analyze functional relationships to explain how a change in one quantity results in a change in another.

7.4 Use patterns and functions to represent and solve problems.

7.5 Describe functions using graphical, numerical, physical, algebraic, and verbal models or representations.

Upon Graduation – End of Grade 12

Students will:

7.1 Describe functions and their inverses using graphical, numerical, physical, algebraic, and verbal mathematical models or representations.

7.2 Analyze the graphs of the families of polynomial, rational, power, exponential, logarithmic, and periodic functions.

7.3 Analyze the effects of parameter changes on the graphs of functions and relations, including translations.

7.4 Model real-world phenomena with a variety of functions.

7.5 Use graphing for parametric equations, three-dimensional equations, and recursive relations.

Mathematics Performance Standards: A Profile of Four Levels

The Mathematics Performance Standards describe students' knowledge, skills, and abilities in the mathematics content area on a continuum from kindergarten through grade twelve. These descriptions provide a picture or profile of student achievement at the four performance levels: advanced, proficient, nearing proficiency, and novice.

Advanced: This level denotes superior performance.
Proficient: This level denotes solid academic performance for each benchmark. Students reaching this level have demonstrated competency over challenging subject matter, including subject-matter knowledge, application of such knowledge to real-world situations, and analytical skills appropriate to the subject matter.
Nearing Proficiency: This level denotes that the student has partial mastery or prerequisite knowledge and skills fundamental for proficient work at each benchmark.
Novice: This level denotes that the student is beginning to attain the prerequisite knowledge and skills that are fundamental for work at each benchmark.

Grade 4 Mathematics
Advanced A fourth-grade student at the advanced level in mathematics demonstrates superior performance. He/she:
(a) demonstrates self-motivation and emerging independence as a learner;
(b) accurately selects and uses problem-solving strategies;
(c) presents well-organized solutions and communicates in ways that exceed requirements;
(d) uses whole numbers accurately and fluently to estimate, compute, and determine whether results are accurate and reasonable;
(e) effectively applies basic algebraic concepts and clearly communicates representations in a variety of ways;
(f) examines relationships of shapes in the physical world and makes generalizations;
(g) selects and accurately uses appropriate tools for measurement;
(h) accurately predicts and makes reasonable decisions based on data; and
(i) articulately and fluently communicates representations, analyzes patterns, and clearly describes relationships, and applies them to varied situations.
Proficient A fourth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she:
(a) selects and effectively uses appropriate problem-solving strategies;
(b) consistently presents organized solutions;
(c) uses whole numbers to estimate, compute, and determine whether results are accurate;
(d) applies basic algebra concepts and consistently communicates representations in a variety of ways;
(e) consistently examines and accurately uses relationships of shapes in the physical world;
(f) determines measurable attributes of objects and selects appropriate tools for measurement;
(g) consistently predicts and makes reasonable decisions based on data; and
(h) consistently uses a variety of patterns and describes their relationships.

Nearing Proficiency A fourth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:
(a) sometimes selects and uses appropriate problem-solving strategies;
(b) sometimes presents organized solutions, but often with limited supporting information;
(c) uses whole numbers to estimate and compute, and results are usually reasonable;
(d) sometimes applies basic algebraic concepts, but seldom communicates representations;
(e) examines some shapes in the physical world, and sometimes sees relationships;
(f) determines measurable attributes of objects, but does not always select appropriate tools for measurement;
(g) often makes inconsistent predictions and inaccurate decisions based on data; and
(h) uses a limited range of patterns, and sometimes describes relationships within those patterns.
Novice A fourth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental at each benchmark in mathematics. He/she:
(a) selects and uses only a few problem-solving strategies;
(b) often presents poorly organized solutions, often without supporting information or explanation;
(c) lacks clarity and coherence when communicating mathematical concepts;
(d) uses whole numbers to estimate and compute, but is frequently inaccurate;
(e) sometimes determines whether results are reasonable;
(f) demonstrates a basic algebraic understanding of concrete and symbolic representations, but often misconceptions are present;
(g) describes, models, and classifies some shapes;
(h) determines some measurable attributes of objects, but often does not select appropriate tools for measurement;
(i) sometimes predicts, but often makes inaccurate decisions based on data; and
(j) recognizes and represents a limited range of patterns and describes relationships within those patterns, but is frequently inaccurate.

Grade 8 Mathematics
Advanced An eighth-grade student at the advanced level in mathematics demonstrates superior performance. He/she:
(a) demonstrates self-motivation and independence as a learner;
(b) is accurate and fluent when applying mathematical processes;
(c) effectively uses multiple strategies and extends concepts to new situations;
(d) explores hypothetical questions and articulates valid arguments;
(e) applies and extends rational numbers, proportionality, and algebraic concepts to solve real and theoretical problems;
(f) applies complex measurement and geometric relationships to hypothetical situations;
(g) consistently makes accurate predictions and decisions based on basic probability and statistics; and
(h) recognizes interconnections within and outside mathematics.
Proficient An eighth-grade student at the proficient level in mathematics demonstrates solid academic performance. He/she:
(a) effectively applies mathematical processes correctly to solve a variety of problems;

(b) applies mathematics in a variety of contexts;
(c) uses rational numbers, proportionality, and algebraic concepts to represent and accurately solve mathematical problems;
(d) consistently and accurately uses complex measurement, geometric relationships, and properties to describe the physical world;
(e) formulates logical arguments using appropriate mathematical ideas; and
(f) consistently makes reasonable predictions and decisions based on basic probability and statistics.

Nearing Proficiency An eighth-grade student at the nearing proficiency level in mathematics demonstrates partial mastery of the prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:
(a) often uses incomplete and incorrect mathematical processes to solve problems, often inaccurately;
(b) communicates mathematical ideas, but often inaccurately;
(c) makes connections, but does not generalize and often his/her arguments lack appropriate supporting mathematical ideas;
(d) sometimes understands and correctly uses numbers, operations, patterns, relations, and functions;
(e) sometimes uses inaccurate or incomplete representations of rational numbers, proportionality, and algebraic concepts to solve mathematical problems;
(f) sometimes has difficulty recognizing complex measurement and geometric relationships and properties which result in inaccurate solutions; and
(g) makes simple predictions and decisions based on basic probability and statistics.

Novice An eighth-grade student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental to each benchmark in mathematics. He/she:
(a) demonstrates limited and incomplete use of mathematical processes;
(b) communicates mathematical ideas, but they are often limited and incomplete;
(c) sometimes uses numbers, operations, patterns, relations, and functions accurately;
(d) makes only immediate, concrete, mathematical connections;
(e) seldom uses algebraic concepts to solve problems; and
(f) makes simple and inconsistent predictions and decisions, often inaccurately, based on data, and seldom recognizes complex measurement, geometric relationships, or properties.

Upon Graduation Mathematics

Advanced A graduating student at the advanced level in mathematics demonstrates superior performance. He/she:
(a) is self-motivated, an independent learner, and extends and connects ideas;
(b) is accurate, articulate, and effective when applying mathematical processes;
(c) effectively uses multiple strategies, extends concepts to new situations, and skillfully communicates the results;
(d) explores hypothetical questions, uses complex reasoning to articulate valid arguments, and constructs proofs;
(e) uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(f) applies complex measurement and geometric and algebraic relationships to model a variety of problems and situations;

(g) consistently makes accurate and reasonable predictions and decisions based on data, probability, and statistics; and
(h) recognizes interconnections within and outside mathematics.

Proficient A graduating student at the proficient level in mathematics demonstrates solid academic performance. He/she:
(a) consistently applies mathematical processes correctly to solve a variety of problems and communicate the results;
(b) applies mathematics in a variety of contexts;
(c) consistently uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(d) uses complex reasoning to formulate logical arguments and proofs using appropriate mathematical ideas;
(e) consistently applies complex measurement and geometric and algebraic relationships to model a variety of problems and situations;
(f) makes reasonable predictions and decisions based on data, probability, and statistics; and
(g) recognizes interconnections within and outside mathematics.

Nearing Proficiency A graduating student at the nearing proficiency level in mathematics demonstrates partial mastery of the
prerequisite knowledge and skills fundamental for proficient-level mathematics. He/she:
(a) applies incomplete and incorrect mathematical processes to solve problems, often inaccurately;
(b) communicates mathematical ideas and sometimes extends them, but often inaccurately;
(c) sometimes understands and uses appropriate technology to apply functions, graphs, and algebraic concepts to solve real and theoretical problems;
(d) sometimes demonstrates difficulty recognizing complex measurement and geometric and algebraic relationships which result in inaccuracies;
(e) sometimes makes predictions and decisions based on data, probability, and statistics, often inaccurately; and
(f) makes connections, but does not generalize or prove them and often his/her arguments lack appropriate supporting mathematical ideas and careful reasoning.

Novice A graduating student at the novice level in mathematics is beginning to attain the prerequisite knowledge and skills that are fundamental at each benchmark in mathematics. He/she:
(a) demonstrates limited and incomplete use of mathematical processes and problem-solving strategies;
(b) often uses limited and incomplete reasoning to formulate logical arguments and communicate mathematical ideas;
(c) makes only concrete, mathematical connections;
(d) seldom uses appropriate technology to apply functions, graphs, and algebraic concepts to solve problems;
(e) recognizes, on a limited basis, complex measurement, geometric relationships, and properties; and
(f) makes some predictions and decisions, on a limited basis, based on data, but seldom recognizes statistical or probability concepts.